

Sec. 14.1:

Functions of Several Variables

What We Will Go Over In Section 14.1

1. Functions of 2 Variables

- a) What is it?
- b) Evaluating
- c) Domain
- d) Range
- e) Graphs
- f) Level Curves

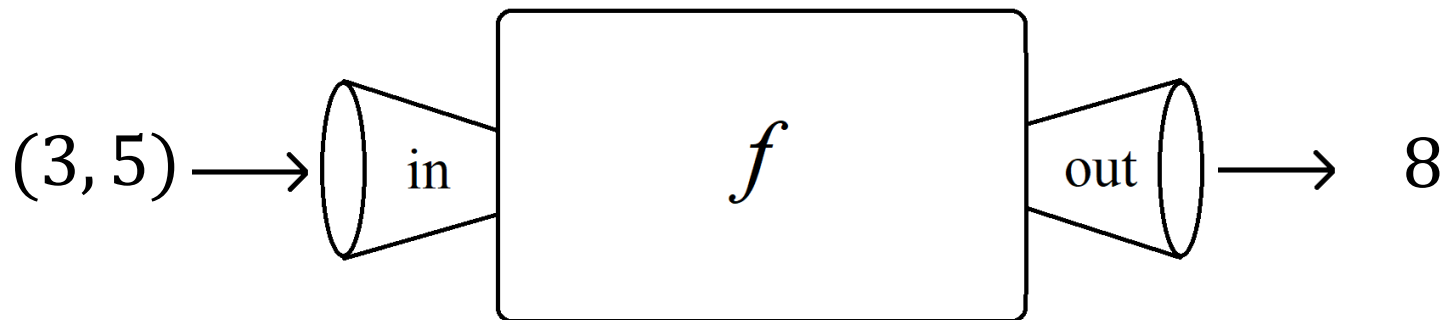
2. Functions of 3 or More Variables

What is a function?

General Functions

Ex: Let f be the function that takes an ordered pair of numbers and returns a single number by the following rule...

The output is the sum of the 2 input numbers



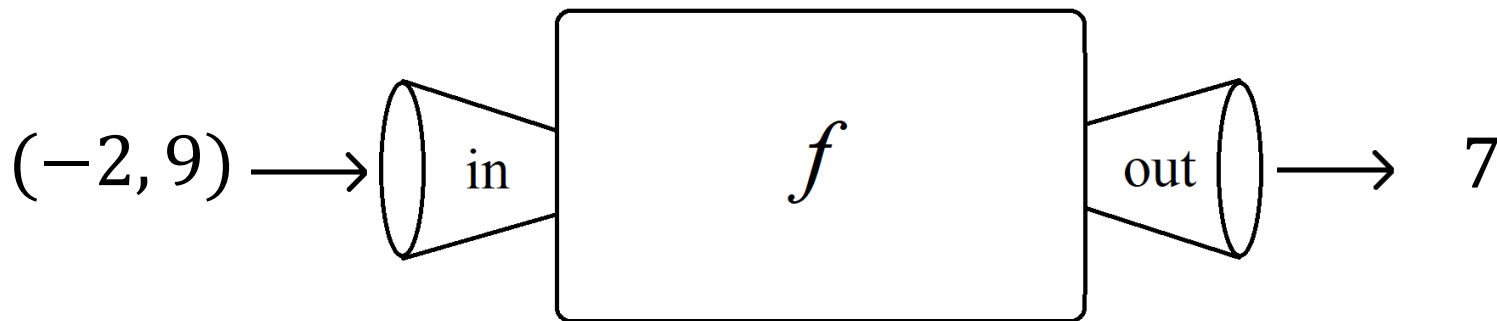
$$f(3, 5) = 8$$

What is a function?

General Functions

Ex: Let f be the function that takes an ordered pair of numbers and returns a single number by the following rule...

The output is the sum of the 2 input numbers



$$f(3, 5) = 8$$

$$f(-2, 9) = 7$$

$$\text{Domain} = \mathbb{R}^2$$

$$\text{Range} = \mathbb{R}$$

What is a function?

General Functions

Ex: Let f be the function that takes an ordered pair of numbers and returns a single number by the following rule...

The output is the sum of the 2 input numbers

Formula: $f(x, y) = x + y$

What is a function?

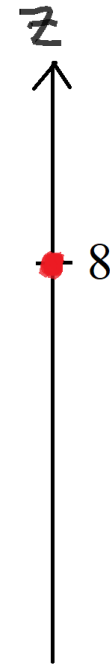
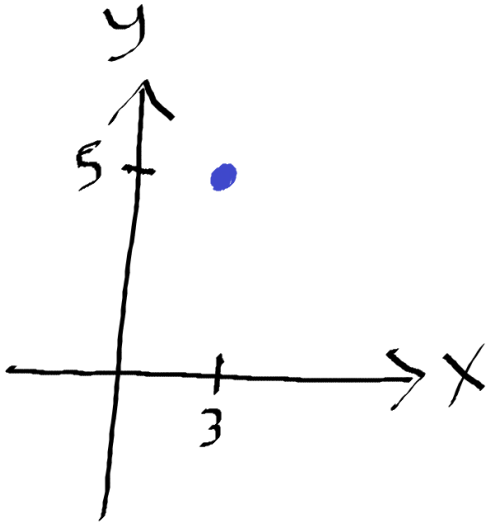
General Functions

Ex: $f(x, y) = x + y$

Domain = $\mathbb{R}^2$

Range = $\mathbb{R}$

$$f(3, 5) = 8$$



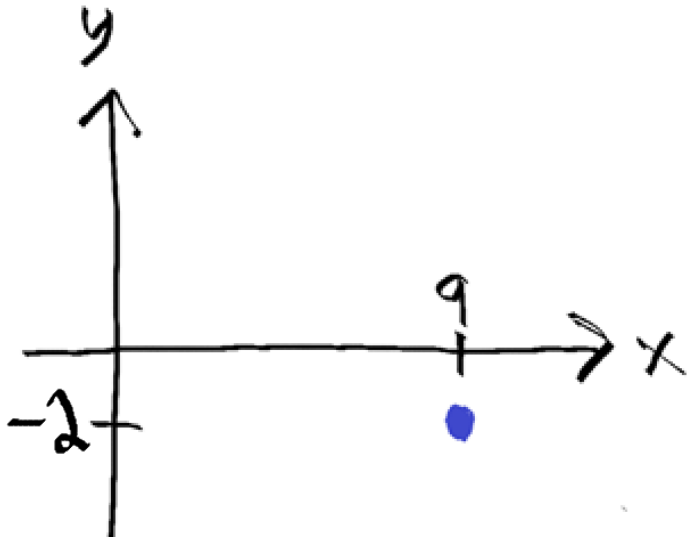
What is a function?

General Functions

Ex: $f(x, y) = x + y$

Domain = $\mathbb{R}^2$

$$f(9, -2) = 7$$



Range = $\mathbb{R}$

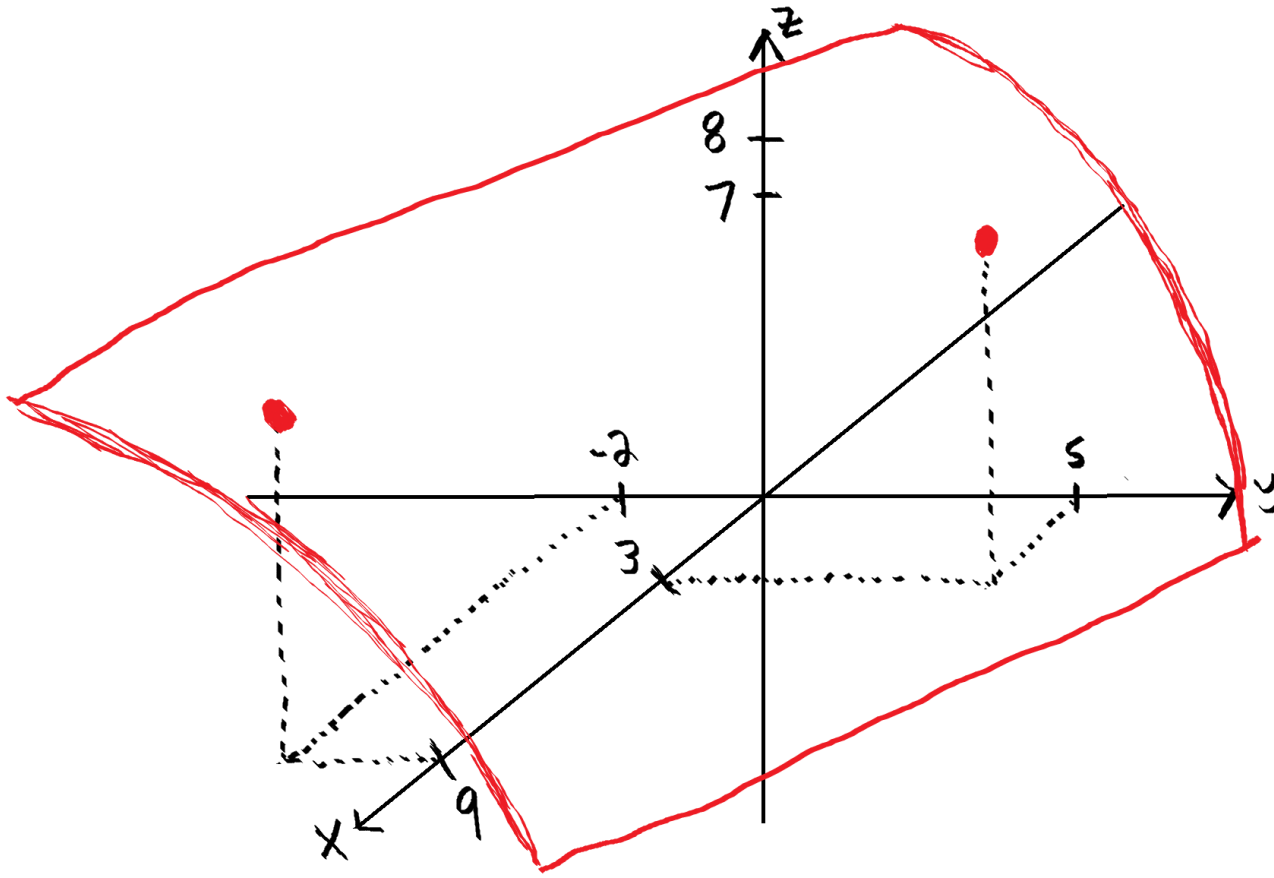


What is a function?

General Functions

Ex: $f(x, y) = x + y$

$$f(3, 5) = 8 \longrightarrow (3, 5, 8)$$
$$f(9, -2) = 7 \longrightarrow (9, -2, 7)$$



1. Functions of 2 Variables

What is it? / Domain / Range

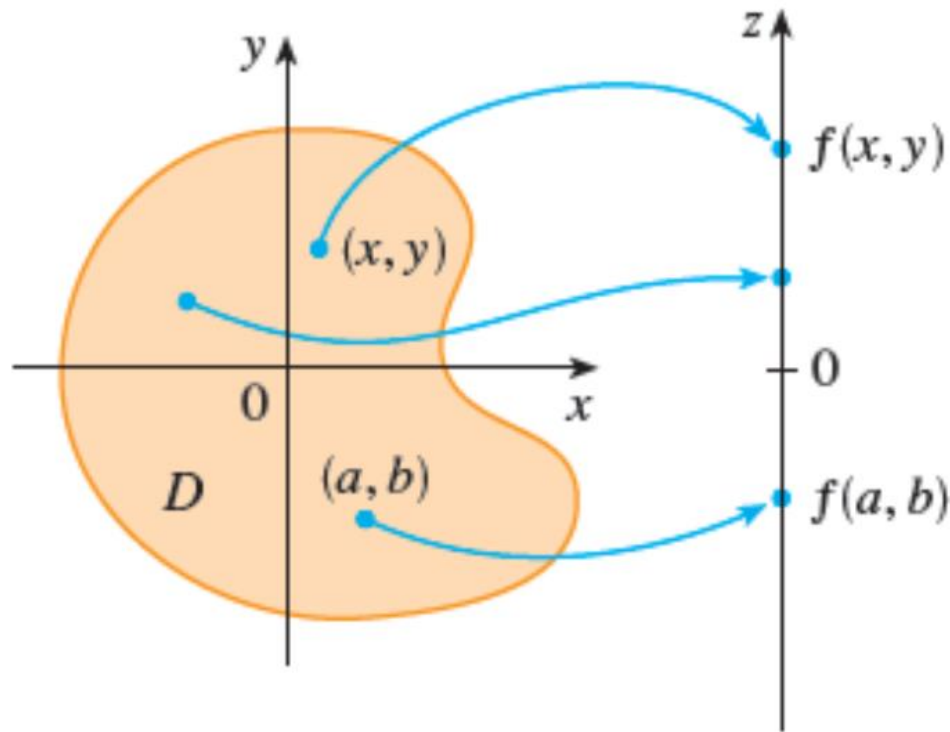
Definition

A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

1. Functions of 2 Variables

What is it?

A 2 variable function is a function whose inputs are ordered pairs and whose outputs are single numbers.



1. Functions of 2 Variables

Domain of a 2 Variable Function

The domain of a function is the set of all possible inputs of the function.

- In some problems, the domain is given to us.
- Otherwise, the domain is the group of ALL inputs that, when plugged in to the function, give a defined output.
- For a 2 variable function, the domain is a subset of $\mathbb{R}^2$.

1. Functions of 2 Variables

Range of a 2 Variable Function

The range of a function is the set of all possible outputs of the function.

- For a 2 variable function, the range is a subset of $\mathbb{R}$.
- In problems where the range is hard to find, consider graphing the function first.

1. Functions of 2 Variables

Ex 1: Let $f(x, y) = \frac{\sin^{-1}(x+y)}{x+1}$.

a) Find $f\left(-\frac{1}{2}, 1\right)$

b) Find $f(2, 5)$

c) Find and sketch the domain of f

1. Functions of 2 Variables

Ex 2: Find and sketch the domain of

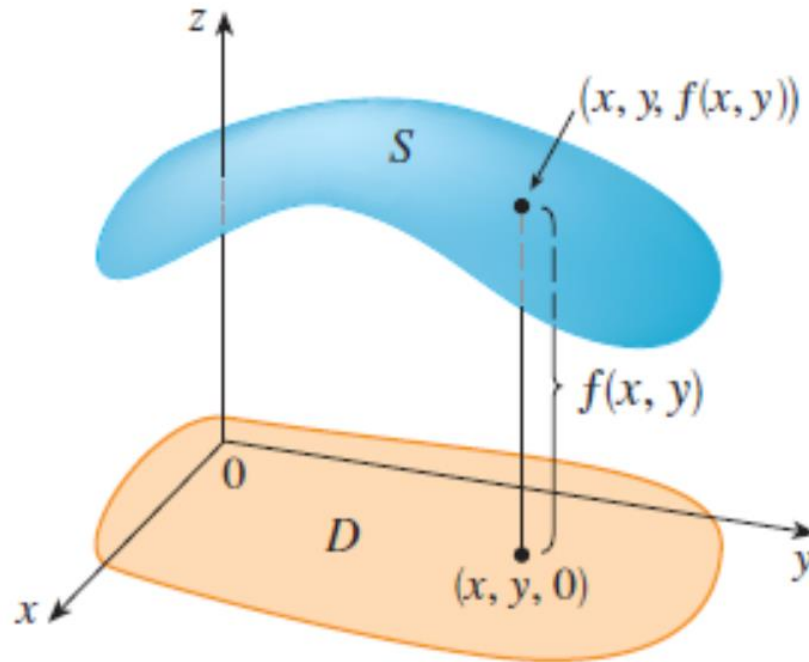
$$g(x, y) = \frac{1}{\ln(y-x^2)} + \sqrt{100 - 25x^2 - 4y^2}.$$

1. Functions of 2 Variables

Graphing a Function of 2 Variables

Definition

If f is a function of two variables with domain D , then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D .



1. Functions of 2 Variables

Ex 3: Let $h(x, y) = \sqrt{9 - x^2 - y^2}$.

- a) Find the domain of h
- b) Find the range of h
- c) Sketch the graph of h

1. Functions of 2 Variables

Ex 4: Sketch the graph of $f(x, y) = 12 - 6x - 4y$.

1. Functions of 2 Variables

Ex 5: Let $g(x, y) = 4x^2 + y^2$.

- a) Find the domain of g
- b) Find the range of g
- c) Sketch the graph of g

1. Functions of 2 Variables

Level Curves and Contour Maps

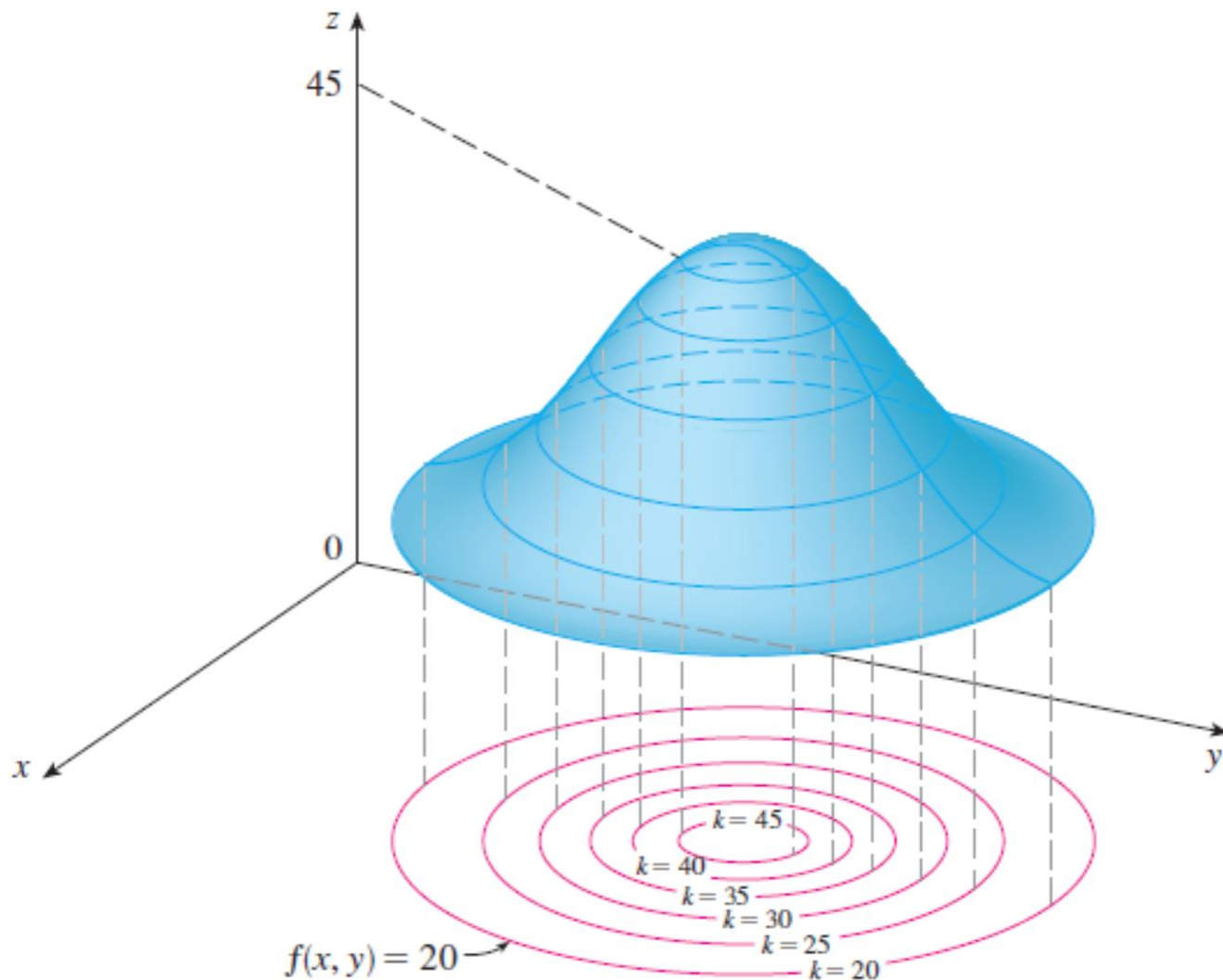
Definition

The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f).

Level curves are just traces parallel to the xy -plane, projected onto the xy -plane.

1. Functions of 2 Variables

Level Curves and Contour Maps



1. Functions of 2 Variables

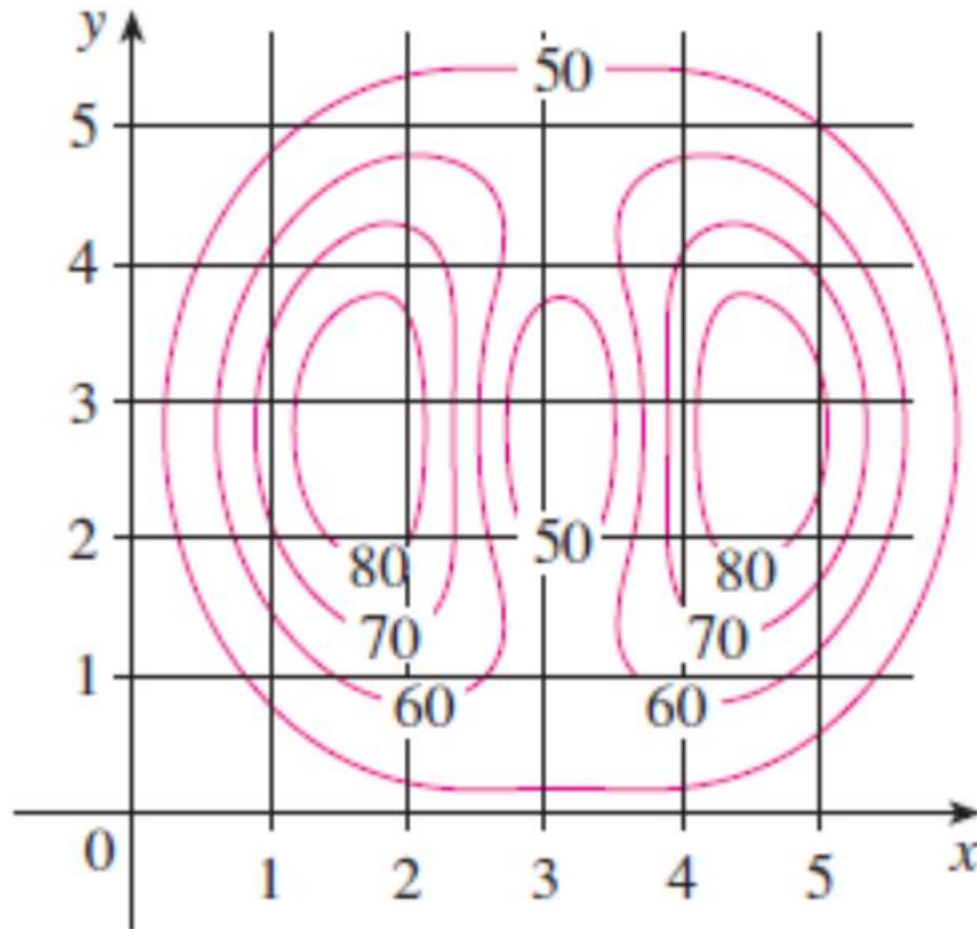
Level Curves and Contour Maps

A contour map is a collection of level curves drawn for different values of k .

- Usually in a contour map, level curves are drawn for evenly spaced out values of k .
- A contour map helps you visualize the graph of the 2 variable equation.

1. Functions of 2 Variables

Ex 6: A contour map for the function f is shown below. Use it to estimate $f(1, 3)$ and $f(4, 5)$.



1. Functions of 2 Variables

Ex 7: Draw a contour map for $f(x, y) = 6 - 3x - 2y$

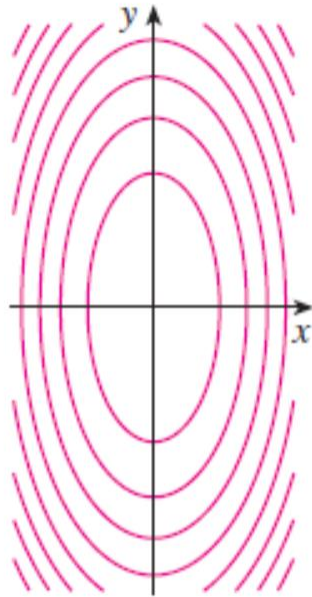
1. Functions of 2 Variables

Ex 8: Draw a contour map for

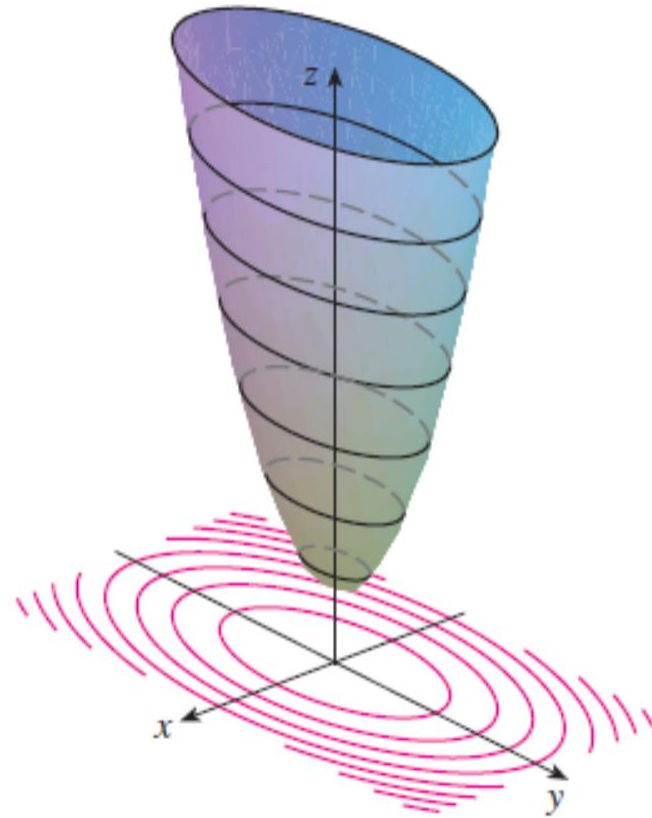
$$g(x, y) = \sqrt{9 - x^2 - y^2}$$

1. Functions of 2 Variables

The graph of $h(x, y) = 4x^2 + y^2 + 1$ is formed by lifting the level curves.



(a) Contour map



(b) Horizontal traces are raised level curves

1. Functions of 2 Variables

Ex 9: Match the function with it's graph.

a. $f(x, y) = \frac{1}{1 + x^2 + y^2}$

b. $f(x, y) = \frac{1}{1 + x^2 y^2}$

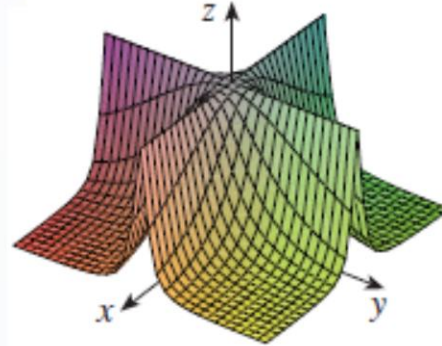
c. $f(x, y) = \ln(x^2 + y^2)$

d. $f(x, y) = \cos \sqrt{x^2 + y^2}$

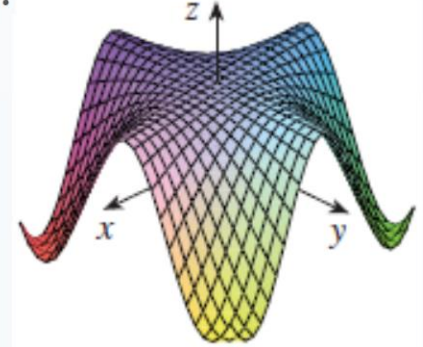
e. $f(x, y) = |xy|$

f. $f(x, y) = \cos(xy)$

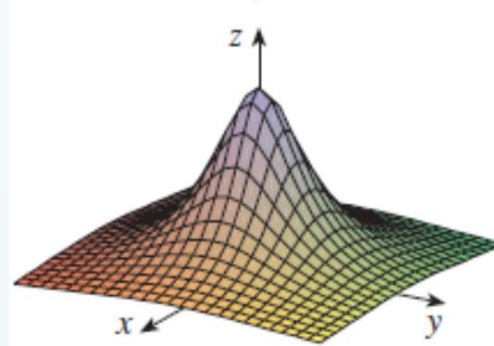
I.



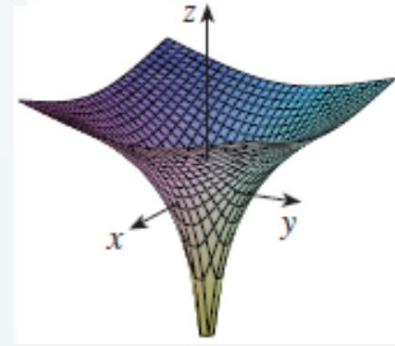
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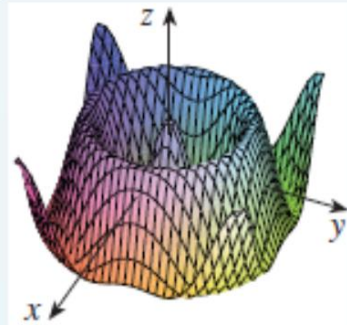
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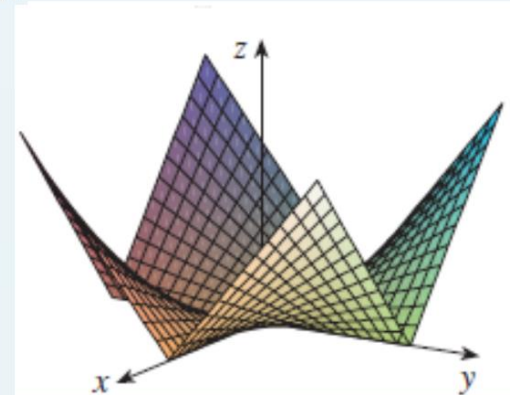
IV.



V.



VI.



2. Functions of 3 or More Variables

A function of 3 or more variables is a rule that assigns to each member of a subset of $\mathbb{R}^n$ a number.

- Inputs are n-tuples
- Outputs are single numbers
- When n is 3 or more, we lose the ability to graph.
- When $n = 3$, the points where $f(x, y, z) = k$ are called level surfaces.

1. Functions of 2 Variables

Ex 10: For the function

$$f(x, y, z) = \ln(z - y) + xysin z$$

- a) Find $f\left(\frac{1}{2}, 3, \frac{\pi}{4}\right)$
- b) The domain of f

1. Functions of 2 Variables

Ex 11: Find the level surfaces of

$$f(x, y, z) = x^2 + y^2 + z^2$$